44 (BCA–1) 1·4

2022

(Held in 2023)

MATHEMATICS–I

Paper : BCA–1·4

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 2×5=10

(a) If $A=\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, find $(A-2I)(A-3I)$.

I is the identity matrix of type 2×2.

(b) Define null matrix and identity matrix.

(c) If $A=\begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$, find BA.

Can we find AB also ?

(d) Find the minor and co-factor of b of the matrix $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$.

(e) Find the value of the determinant of the matrix $A = \begin{bmatrix} 4 & 7 & 8 \\ -9 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$.

2. (a) If $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$

verify that $(AB)' = B'A'$. 3

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

show that $A^2 - 4A - 5I = 0$. 3

(c) Prove that every square matrix can be expressed as a sum of symmetric and skew-symmetric matrices. 3

(d) Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

3

(e) Find the rank of matrix

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}.$$

3

3. Determine the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

5

4. (a) Find the argument and modulus of the complex number $1 + i\sqrt{3}$. 2+2=4

(b) Express the polar form $1 + i$. 2

(c) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in G.P. 3

(d) Find the quadratic equation whose roots are 3 and 4. 2

5. *(a)* Evaluate : ***(any three)*** 3×3=9

(i) $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$

(ii) $\lim_{x \to \infty} \frac{x^2 + x - 5}{x^2 + 2x + 3}$

(iii) $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

(iv) $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$

(v) $\lim_{x \to \pi/2} \frac{2x - \pi}{\cos x}$

(b) Find the derivatives of the function $f(x) = x^2$ by first principle of derivatives. 3

(c) Find $\frac{dy}{dx}$ for $y = x^2 - 3x + 2$. 2

(d) Find $\frac{dy}{dx}$ for the following : ***(any two)*** 3×2=6

(i) $\frac{x^2 + 3}{x - 2}$

(ii) $e^{2x} + \sin x - 4x^2 + 5$

(iii) $x^2 \cos x + 3e^{-2x}$

6. (a) State and give Geometrical interpretation of Lagrange's Mean value theorem or Rolle's theorem. 2+3=5

(b) Find the maximum or minimum values of $f(x) = x^{1/x}$ 5

Or

$f(x) = \sin x + \cos x$

(c) Evaluate : $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$ 5

Or

$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(d) Verify Rolle's theorem for $f(x) = x^2 - 3x + 2$ in $[1, 2]$. 4